NOETHERIAN RINGS WHOSE INJECTIVE HULLS OF SIMPLE MODULES
ARE LOCALLY ARTINIAN

PAULA CARVALHO, CHRISTIAN LOMP

The Jacobson’s conjecture is an open problem in ring theory and asks whether the intersection of the
powers of the Jacobson radical of a two-sided Noetherian ring is zero. Jategaonkar answered the conjecture
in the affirmative for a Noetherian ring $R$ under an additional assumption (called FBN) which in particular
implies that any finitely generated essential extension of a simple left $R$-module is Artinian. The latter
condition, denoted by $(\Diamond)$, is a sufficient condition for a positive answer to the Jacobson’s conjecture.

In these two talks we will discuss the question which Noetherian algebras do or do not satisfy condition
$(\Diamond)$. In the first part we will review the history of property $(\Diamond)$ and will explicitly characterize those
enveloping algebras of finite dimensional complex nilpotent Lie superalgebras that do satisfy $(\Diamond)$ while in the
second part we will consider differential operator rings $R[x;d]$ with $R$ a commutative Noetherian ring and $d$
is a derivation as well as so-called, down-up algebras.

Down-up algebras were introduced by G.Benkart and T.Roby (J.Algebra 1998) and form a three-paramenter
family of associative algebras $A(\alpha,\beta,\gamma)$ in two generators $u$ and $d$ over some field $K$ subject to two cubic
relations that depend upon the scalar parameters $\alpha, \beta$ and $\gamma$. Using the fact that when $\gamma = 0$ (resp $\gamma = 1$) the
quantum plane, (resp. the quantized Weyl algebra) is an image of $A = A(\alpha, \beta, \gamma)$, we will exhibit examples
of down-up algebras that do not satisfy property $(\Diamond)$. 