A counterexample to the first Zassenhaus conjecture

Leo Margolis

Abstract: In 1974 H. Zassenhaus conjectured that any unit of finite order in the integral group ring $\mathbb{Z}G$ of a finite group G is conjugate in the rational group algebra of G to a trivial unit of the form $\pm g$, for some $g \in G$. This was considered to be the strongest possible general statement about torsion units in integral group rings of finite groups. The conjecture had been proven for several classes of groups such as nilpotent groups or cyclic-by-abelian groups.

I will present a metabelian counterexample found in collaboration with Florian Eisele and the techniques underlying the construction. Proving or disproving the conjecture for a certain group G is equivalent to classifying a class of $\mathbb{Z}(G \times C_n)$-modules where n is the order of the unit in question. As the module theory of integral group rings is usually to complicated to allow a direct approach first modules of the rational group algebra of G are considered which can be investigated using character theory. Then modules of p-adic group rings corresponding to a potential counterexamples can be explicitly constructed under certain restrictions on G. Finally to show that this implies the existence of a counterexample to the conjecture of Zassenhaus one has to consider the genus class group. These general observations are then sufficient to verify a class of counterexamples by elementary character and group theoretic calculations.