BIRKHOFF-PIERCE PROBLEM AND WEINBERG CONJECTURE OF LATTICE-ORDERED MATRIX RINGS

Abstract. In 1956, G. Birkhoff and R. Pierce raised two questions in their famous paper "Lattice-ordered rings". One question is now known as "Pierce-Birkhoff Conjecture" which states that a piecewise polynomial function from $\mathbb{R}^n$ to $\mathbb{R}$ is the maximum of minima of a finite family of polynomials. The topic of this talk is the other one which is much less known: can the complex numbers be lattice-ordered as a field? It is well-known and easy to prove that any field with -1 a sum of squares can't be totally ordered, so the question just says whether we can equip the complex numbers with an ordering better enough. In 1966, E. Weinberg made a conjecture, now called the Weinberg Conjecture, toward this problem, which states that the full matrix ring $M_n(\mathbb{Q})$ over rationals $\mathbb{Q}$ has up to isomorphism unique lattice-ordering under which the identity matrix being positive. Unfortunately, Both Birkhoff-Pierce problem and Weinberg conjecture remain open. I'll talk about the story.