# Annual conference of the DFG priority programme in representation theory, SPP 1388

March 25 to 28, 2013, Evangelische Akademie Bad Boll, Germany

Titles and Abstracts

**All talks will take place at the Festsaal of the Evangelische Akademie Bad Boll**

## MONDAY, MARCH 25

14.00-14:50 **Aaron Lauda** (University of Southern California)
*Knot invariants and diagrammatic descriptions of representation categories via Howe duality*
**Abstract**: It is a well understood story that one can extract link invariants from quantum groups associated to simple Lie algebras. These invariants are called Reshetikhin-Turaev invariants and the famous Jones polynomial is the simplest example. Kauffman showed that the Jones polynomial could be described very simply by replacing crossings in a knot diagrams by various smoothings. In this talk we will explain Cautis-Kamnitzer-Licata's simple new approach to understanding these invariants using basic representation theory and the quantum Weyl group action. Their approach is based on a version of Howe duality for exterior algebras called skew-Howe duality. Even the graphical (or skein theory) description of these invariants can be recovered in an elementary way from this data. The advantage of this approach is that it suggests a 'categorification' where knot homology theories arise in an elementary way from higher representation theory and the structure of categorified quantum groups.

15:00-15:40 **Susanne Danz** (University of Kaiserslautern)
*Twisted category algebras and quasi-heredity*
**Abstract**: In this talk we shall consider twisted category algebras over fields of characteristic 0. The underlying category will always be finite and will have an additional property, which is called 'split'. The multiplication in such an algebra is essentially induced by the composition of morphisms in the category. Prominent examples of twisted category algebras are various classes of diagram algebras (for suitable parameters) such as Brauer algebras, Temperley–Lieb algebras, or partition algebras. Twisted category algebras also arise in connection with double Burnside rings and biset functors.
We shall show that a twisted split category algebra in characteristic 0 is quasi-hereditary, that is, the corresponding module category is a highest weight category. Moreover, we shall give an explicit description of its standard modules with respect to a particular partial order on the set of isomorphism classes of simple modules. This provides, in particular, a unified proof of the known fact that the aforementioned diagram algebras are quasi-hereditary.
This is joint work with Robert Boltje.

16:20-16:45 **Hans Franzen** (University of Wuppertal)
*Cohomology Rings of Fine Quiver Moduli are tautologically presented*
**Abstract**: We show that cohomology rings of fine quiver moduli spaces can be presented by relations of representation-theoretic origin and illustrate this with several classes of classical examples.

16:50-17:30 **Philipp Lampe** (University of Bielefeld)
*What do cluster structures tell you about an algebra?*
**Abstract**: Some commutative algebras also carry the structure of a cluster algebra, and when this happens, the same algebra may be endowed with possibly many cluster algebra structures. In this talk, we wish to discuss the effect of cluster structures on a given commutative algebra.
Previous work in this area has concerned the construction of linear bases for various cluster algebras. In contrast, our approach seeks to study more explicitly the algebraic properties of a given cluster algebra. This question

may be of interest to representation theorists, because in the categorification of cluster algebras via Buan-Marsh-Reineke-Reiten-Todorov's cluster categories, such algebraic properties play a crucial role.
In particular, we wish to address the following questions: When is a cluster algebra a unique factorization domain? What are its irreducible elements? What is its divisor class group?

17:40-18:30 **Pham Tiep** (University of Arizona)
*Representations of finite groups: Conjectures, reductions, and applications (I)*
**Abstract**: We will discuss some basic problems in representation theory of finite groups, including some long-standing conjectures of Alperin, Brauer, and others. A possible approach to some of these problems is to use the classification of finite simple groups to reduce the problem in consideration to some, more specific, questions about simple groups. We will describe recent progress on reduction theorems in this direction. We will also outline applications of these results to various problems in group theory and algebraic geometry.

## TUESDAY, MARCH 26

09:00-09:50 **Geordie Williamson** (Max Planck Institute for Mathematics, Bonn)
*The Hodge Theory of Soergel bimodules (I)*
**Abstract**: I will explain joint work with Ben Elias which gives an algebraic proof of the Kazhdan-Lusztig conjecture, a character formula for simple highest weight modules over a semi-simple Lie algebra, as well as the Kazhdan-Lusztig positivity conjectures. In the proof Soergel bimodules, as well as ideas from higher algebra, categorification and Hodge theory (in particular work of de Cataldo and Migliorini) play a crucial role.

10:00-10:15 **Wolfgang Soergel** (University of Freiburg)
*What is the Lusztig Conjecture?*

10:50-11:30 **Benjamin Sambale** (University of Jena)
*On Loewy lengths of blocks*
**Abstract**: I present a joint paper with B. Külshammer and S. Koshitani about Loewy lengths of blocks of finite groups. One of the main results states that the Loewy length grows with the defect of the block.

11:40-12:30 **Maud De Visscher** (City University, London)
*Representation theory of the partition algebra*
**Abstract**: The partition algebra was introduced by P. Martin. Over the complex numbers, it satisfies a double centraliser property with the symmetric group via an action on tensor space. In the first part of this talk I will review its representation theory over the complex numbers and investigate some consequences for the symmetric group (joint work with C. Bowman and R. Orellana). In the second part of the talk I will present some recent results on the representation theory of the partition algebra in positive characteristic (joint work with C. Bowman and O. King).

14:30-15:20 **Britta Späth** (University of Kaiserslautern)
*An approach to global/local conjectures in the representation theory of finite groups*
**Abstract**: Much of the recent work in the representation theory of finite groups is centered around the global/local conjectures, notably the conjectures from Alperin, Brauer and McKay. An underlying idea of these conjectures is that certain aspects of the representation theory of a finite group should be determined "locally", that is, by the representation theory of normalisers of certain $p$-subgroups. The Classification of Finite Simple Groups enables to attack several of these conjectures by reducing them to statements about simple groups. Furthermore for some of the simple groups those statements could already be checked.

15:30-15:55 **Julia Worch** (University of Kiel)
*Equal images modules and Auslander- Reiten theory for generalized Beilinson algebras*

**Abstract**: Addressing representations of finite group schemes over fields of positive characteristic, Carlson, Friedlander, Pevtsova and Suslin have introduced the categories of modules of constant Jordan type and of modules with the equal images property, respectively. For an elementary abelian $p$-group $E_r$ of rank $r \geq 2$, we use a faithful exact functor $F : \mathrm{mod}\, B(n;r) \to \mathrm{mod}\, E_r$, where $B(n;r)$ denotes the generalized Beilinson algebra on $n \geq 2$ vertices, in order to define classes of $B(n;r)$-modules that correspond to the ones mentioned above. Interpreting $B(n;r)$ as an iterated one-point extension of the $r$-Kronecker algebra by duals of equal images modules, we exploit earlier work on Auslander-Reiten components of $B(2;r)$ to obtain information concerning the occurrence of the corresponding modules within the Auslander-Reiten quiver $\Gamma(n;r)$ of $B(n;r)$ for $n \geq 3$. In particular, we show that certain modules with the equal images property, the so-called generalized $W$-modules, belong to $\mathbb{Z}A_\infty$-components of $\Gamma(n;r)$. Moreover, these components entirely consist of modules of constant Jordan type.

16:50-17:15 **Wassilij Gnedin** (University of Köln)
*Maximal Cohen-Macaulay modules over some non-reduced Curve Singularities*
**Abstract**: In this talk, we give a summary of our joint work with Igor Burban (arXiv:1301.3305). We prove that the non-reduced curve singularities $k[[x,y,z]]/(xy,y^q-z^2)$ have tame Cohen- Macaulay representation type. For the singularity $k[[x,y,z]]/(xy,z^2)$ we give an explicit description of all indecomposable maximal Cohen-Macaulay modules. The last result is applied to construct explicit families of indecomposable matrix factorizations of $(xy)^2$.

17:20-17:45 **Phillip Linke** (University of Bielefeld)
*Computational approach to the Artinian conjecture*
**Abstract**: What is generic representation theory? When looking at the category $\mathcal{F}_q = \mathrm{Func}(\mathrm{mod}\ \mathbb{F}_q, \mathrm{Mod}\ \mathbb{F}_q)$ we obtain that a functor $F \in \mathcal{F}_q$ generically gives rise to representations of $\mathrm{GL}(V)$ for all $V \in \mathrm{mod}\mathbb{F}_q$. By the Yoneda-lemma we know how certain projectives in $\mathcal{F}_q$ look like. For each $V \in \mathrm{mod}\mathbb{F}_q$, $\mathrm{Hom}(V,-)$ is projective. Such a projective is called a standard projective. It turns out that these standard projective even generate the whole category.
In the 1980s Lionel Schwartz conjectured that all the standard projectives would be noetherian. If true this would imply that every finitely generated functor in $\mathcal{F}_q$ admits a projective resolution by finitely generated projectives. There are partial results that back up this conjecture but no solution so far.
In the talk we will not reach quite as far. The aim is to give an idea why the category $\mathcal{F}_q$ is at least coherent. That means that every finitely presented functor admits a resolution by finitely generated projectives. To get to this goal we will use certain combinatorial properties of the dimension function $\phi(F,n) = \dim_{\mathbb{F}_q} F(\mathbb{F}_q^n)$ for a functor $F \in \mathcal{F}_q$.

17:50-18:30 **Ivan Penkov** (Jacobs University Bremen)
*Primitive ideals in $U(sl(\infty))$, $U(so(\infty))$, $U(sp(\infty))$*
**Abstract**: I will review recent work on the primitive ideals corresponding to the finitary Lie algebras $sl(\infty)$, $so(\infty)$, $sp(\infty)$, and will state how far are we from a full classification. The theory of primitive ideals in the infinite-dimensional case differs notably from the finite-dimensional case, so the passage to infinity is by no means obvious.

## WEDNESDAY, MARCH 27

09:00-09:50 **Aaron Lauda** (University of Southern California)
*Categorified knot invariants from categorified Howe duality*
**Abstract**: Traditional representation theory of Lie algebras studies actions of the Lie algebra on vector spaces. Categorical representation theory studies actions of Lie algebras on categories, with Lie algebra generators acting by functors, and equations between elements lifting to isomorphisms of functors. Categorified quantum groups govern what kinds of natural transformations one can expect between these functors. It turns out that this higher structure can be encoded in a convenient graphical calculus. We will explain how categorified quantum groups and a categorification of the quantum Weyl group action can be used to categorify quantum link invariants.

10:00-10:15 **Sven Meinhardt** (University of Wuppertal)
*What is a Donaldson-Thomas invariant?*
**Abstract**: Donaldson-Thomas theory is a new way to think about moduli spaces for certain objects arising in representation theory, algebraic geometry and symplectic geometry. I will give a very gentle introduction and sketch the main ideas focusing on representation theory.

10:50-11:30 **Olivier Dudas** (University Paris Diderot)
*Finding PIM's for finite groups of Lie type*
**Abstract**: Given a finite group of Lie type $G$, such as $GL_n(q)$, $Sp_{2n}(q)$,... we are interested in the representation theory of $G$ over a field of characteristic $l$ (coprime to $q$). Part of the information is contained in the decomposition matrix of $G$, which encodes the characters of the projective indecomposable modules. I will describe how one can use the mod-$l$ cohomology of Deligne-Lusztig varieties to construct such modules and compute their characters, leading to new decomposition matrices for groups of small rank.

11:40-12:30 **Pham Tiep** (University of Arizona)
*Representations of finite groups: Conjectures, reductions, and applications (II)*

14:30-15:20 **Geordie Williamson** (Max Planck Institute for Mathematics, Bonn)
*The Hodge theory of Soergel bimodules (II)*

15:30-15:55 **Oliver Straser** (University of Freiburg)
*Koszul Duality and Geometric Satake*
**Abstract**: Koszul Duality gives a geometric description of the category of Harish Chandra modules for $SL_2$ in terms of the equivariant derived category of the geometric parameter space. We will give a geometric construction of the functor "tensoration with finite dimensional representations" looking very similar to the convolution of perverse sheaves on the Affine Grassmannian. As an application we get a graded lift of the tensor functor.

16:40-17:05 **Frederik Marks** (University of Stuttgart)
*From ring epimorphisms to universal localisations*
**Abstract**: Ring epimorphisms are relevant to study certain subcategories of a fixed module category or of its derived category. In this talk, we provide sufficient conditions for a ring epimorphism to be a universal localisation, in the sense of Cohn and Schofield. Furthermore, we consider recollements induced by some homological ring epimorphisms and investigate whether they yield recollements of derived module categories. This is joint work with Jorge Vitória.

17:10-17:35 **Markus Jedlitschky** (University of Stuttgart)
*Decomposing Supercharacters of unipotent orthogonal groups into orbit modules*
**Abstract**: Let $p$ be a prime and $q$ a power of that prime. We call the $p$-Sylow subgroup of the full linear group over the field of $q$ elements 'unipotent linear group' and the $p$-Sylow subgroup $U$ of the orthogonal group (of type $D$) 'unipotent orthogonal group'. André and Neto constructed a supercharacter theory (in the sense of Diaconis and Isaacs) for the unipotent orthogonal groups $U$ by restricting certain supercharacters of the unipotent linear group to $U$. We define a new monomial basis for the regular representation of the unipotent orthogonal group and decompose the representations affording the supercharacters of $U$ into orbit modules.

17:40-18:20 **Andrew Hubery** (University of Leeds/University of Bielefeld)
*Irreducible Components of Quiver Grassmannians*
**Abstract**: We develop some general scheme-theoretic results which can be applied to compute irreducible components of schemes arising from representation theory, including representation spaces and quiver Grassmannians

or more generally quiver flag varieties. In particular, we prove a decomposition theorem for irreducible components, analogous to the Krull-Remak-Schmidt Theorem for modules, and extending the work of Crawley-Boevey and Schröer for the representation varieties.

## 1. THURSDAY, MARCH 28

09:00-09:50 **Martin Kalck** (University of Bielefeld)
*Relative singularity categories*
**Abstract**: We discuss the relations between two triangulated categories associated with a Gorenstein singularity $R$: the singularity category of Buchweitz and Orlov and the relative singularity category, which is associated with a (non-commutative) resolution $A$ of $R$ and was studied by Chen, Thanhoffer de Völcsey & Van den Bergh and Burban & Kalck. This talk is based on joint work with Dong Yang.

10:00-10:15 **Peter Littelmann** (University of Köln)
*What are standard monomials?*

10:50-11:30 **Igor Burban** (University of Köln)
*Matrix problems, vector bundles on curves of genus one and the classical Yang-Baxter equation*
**Abstract**: In my talk (partially based on a joint work with Thilo Henrich arXiv:1202.5738) I shall explain a connection between the study of special classes of coherent sheaves on elliptic curves and their degenerations with solutions of the classical Yang-Baxter equation. In particular, I shall show how Atiyah's classification of indecomposable vector bundles on elliptic curves is related with elliptic r-matrices of Belavin. In the case of a cuspidal or tachnode cubic curve, this theory combined with the technique of matrix problems (representations of bocses) and Frobenius Lie algebras, leads to some distinguished classes of rational r-matrices.

11:40-12:30 **Chris Schommer-Pries** (Max-Planck Institute for Mathematics, Bonn)
*The structure of tensor categories via topology and higher categories*
**Abstract**: Fusion tensor categories arise in many areas of mathematics: as representation categories for finite quantum groups, certain Hopf algebras, and loop groups; as the 'basic invariants' of subfactors of von Neumann algebras in the theory of operator algebras; and also in the study of conformal field theory. Fusion tensor categories have a rich and fascinating structure. The goal of this talk will be to describe how higher categories allow this structure to be understood and explained using 3 dimensional topology and 3 dimensional topological field theory. There are also connections to Jacob Lurie's work on the cobordism hypothesis. This is joint work with Christopher Douglas and Noah Snyder.